MIDTERM 1 STUDY GUIDE

PEYAM RYAN TABRIZIAN

Know how to:

- Given a graph or given a formula, find values of a function, and solve equations such as: Find x such that $f(x) = 2$ (1.1.1)
- Determine if a graph is a graph of a function (1.1.6)
- Sketch graphs of functions representing real-life situations (1.1.14, 1.1.18)
- Find domains and ranges of functions, given a graph or given a formula (1.1.30, 1.1.32, Quiz 1.1-2)
- Solve word problems (1.1.57, 1.3.55, Quiz 1.5)
- Know how to draw graphs of linear functions, power functions (e.g. x^3 or $\sqrt{x}$), and exponential functions (e.g. 3^x) (1.2.4)
- Use the above functions in word problems (1.2.15)
- Graph new functions from old ones, e.g. given f, graph $f(-x)$ (1.3.5, 1.3.6)
- Explain, for example, how you can get the graph of $-f(x+2)+3$ given the graph of f (1.3.2, 1.3.3, Quiz 1.3)
- Compose, add, multiply, and divide functions and find their domains (1.3.33, 1.3.39)
- Compositions represent in real-life situations (1.3.55)
- Find domains of functions involving e^x, e.g. Find the domain of $\frac{e^x}{1+e^x}$ (1.5.15)
- Determine whether a function is one-to-one, given its graph (1.6.6), or given a formula (1.6.9, 1.6.10)
- Find the inverse of a function, given its graph, i.e. reflect about the line $y = x$ (1.6.29, 1.6.30)
- Find the inverse of a function, given a formula (1.6.25, 1.6.26)
- Know what $f^{-1}(4)$ actually means (1.6.17)
- Do computations with ln and logs, and simplifying expressions involving ln and logs (1.6.33, 1.6.36, 1.6.39)
- Find average velocities of a function, given a table or given a formula, and estimate instantaneous velocities (2.1.6, 2.1.7)
- Find the limit of a function at a point or at infinity (or say that it does not exist) and vertical/horizontal asymptotes of a function **given its graph** (2.2.7, 2.2.9, 2.6.3)
- Sketch the graph of a function with given limits (2.2.15, 2.6.7)
- Given 2 graphs of f and g, finding limits of $f + g$, $f \times g$, etc. (2.3.2)
- **Evaluating a limit of a function at a point (or showing that it does not exist), given its equation**:
  - By substituting into the expression (2.3.3, 2.3.5)
  - By noticing, for example, that it's of the form $\frac{1}{0^+}$ (and hence it's $+\infty$) (2.2.25, 2.2.28)

Date: Friday, September 17th, 2010.

  - By noticing that the left-hand limit and the right-hand limit are equal, or not equal, if the limit does not exist (2.3.39, 2.3.40, 2.3.45) **Good for piecewise-defined functions!**
  - By factoring the numerator/denominator, and by 'canceling out' (2.3.11, 2.3.15, also look at 2.3.61)
  - By multiplying numerator and denominator by $\sqrt{a} - b$, whenever you see something involving $\sqrt{a} + b$ (2.3.21, 2.3.22, 2.3.23, 2.3.30, 2.3.60)
  - By using the squeeze theorem (2.3.37, 2.3.38)
- **Evaluating a limit of a function at infinity (or showing that it does not exist) and stating its asymptotes, given its equation**:
  - By substituting into the expression (2.6.15)
  - By factoring out the highest power of the numerator, and the highest power of the denominator (2.6.16, 2.6.19, 2.6.21, also 2.6.33), or simply the highest power of the expression (2.6.31)
  - By multiplying numerator and denominator by $\sqrt{a} - b$, whenever you see something involving $\sqrt{a} + b$ (2.6.25, 2.6.26, 2.6.27)
  - By factoring out the highest power of the square root, when the preceding method fails (2.6.23, 2.6.24)
  - By noticing that the function is bigger than or smaller than a familiar function whose limit you know (2.6.30)
- **Finding limits rigorously, using an $\epsilon - \delta$-argument** (2.4.19, 2.4.22, 2.4.29, 2.4.30, 2.4.31, 2.4.32, 2.3.36, 2.3.37)
- Find **left-hand-side and right-hand-side** limits rigorously, using an epsilon-delta argument (2.4.28)
- Find **infinite** limits rigorously, using an epsilon-delta argument (2.4.42, 2.4.44)
- Find limits **at infinity** rigorously, using an epsilon-delta argument (2.6.65, 2.6.67) (This includes infinite limits at infinity!) **Note:** Be careful! Sometimes you may have a problem that does not involve ϵ explicitly! (look at 2.6.13, 2.6.14)
- Given a graph, state the numbers at which the function is continuous or not (2.5.3)
- Given a formula, show that a function is continuous at a point (2.5.43)
- Given a formula, find the numbers at which a function is discontinuous (2.5.37, 2.5.39)
- Using the intermediate value theorem to show that an equation has a root, or that two functions are equal at a point (2.5.47, 2.5.51)
- Solving word problems using the intermediate value theorem (2.5.65)
- Calculate the derivative of a given function at a given point, using the definition of a derivative (2.7.25, 2.7.27, 2.7.30)
- Recognize a certain limit as a derivative of a function (2.7.31, 2.7.34, 2.7.36)
- Find the equation of the tangent line to the graph of a given function at a given point (2.7.10)
- Know what a derivative means in real life, in particular find the instantaneous velocity of a particle at a given time (2.7.37, 2.7.38, 2.7.46)

Also, know how to define the following terms / state the following theorems (remember that for functions, you'll need to state the domain and the codomain of that function!):

- Function
- Domain of f
- Range of f
- Absolute Value Function
- Increasing/Decreasing
- Vertical line test
- e
- 2^x (more generally a^x)
- $f \circ g$ (f composed with g)
- Inverse function
- $ln(x)$ (more generally $log_a(x)$)
- $log_r(2)$ (just say it's the number y such that $r^y = 2$)
- $\lim_{x\to a} f(x) = L$ (the rigorous definition), as well as its variants $\lim_{x\to a^+} f(x) = L$, $\lim_{x\to a} f(x) = +/-\infty$, and $\lim_{x\to +/-\infty} f(x) = L$, $\lim_{x\to +/-\infty} f(x) = +/-\infty$
- Vertical/Horizontal asymptote
- The Squeeze Theorem
- f is continuous at a (and its variant with left/right-continuous)
- f is continuous on an interval I
- The Intermediate Value Theorem
- The derivative of f at a, i.e. $f'(a)$ (both definitions: the limit-definition, and the tangent-line definition)
- The tangent line to $y = f(x)$ at $P = (a, f(a))$